Mock 6 (Week 5-8)

**Due date for this assignment: 2025-08-02, 23:59 IST.**

**You may submit any number of times before the due date. The final submission will be considered for grading.**

***3 points***

Simplify the expression (axay)(x+y−z).(ayaz)(y+z−x).(azax)(z+x−y)(*ayax*​)(*x*+*y*−*z*).(*azay*​)(*y*+*z*−*x*).(*axaz*​)(*z*+*x*−*y*)

ax+y+z*ax*+*y*+*z*

ax2+y2+z2−xy−yz−zx*ax*2+*y*2+*z*2−*xy*−*yz*−*zx*

1

a*a*

***5 points***

Which of the following statements are correct?

The functions f(x)=−ln(x)*f*(*x*)=−*ln*(*x*)​ and g(x)=ex2*g*(*x*)=*ex*2 are inverses to each other.

The domain of the real-valued function f(x)=ex2−8x−1*f*(*x*)=*ex*2−8*x*−1​ is (−∞,0]∪[8,∞)(−∞,0]∪[8,∞).

The line x=3*x*=3 is a vertical asymptote of the function f(x)=ln(x2+5x−24)*f*(*x*)=*ln*(*x*2+5*x*−24).

f*f* may be continuous at the point x=a*x*=*a* even if f*f* is not differentiable at a point x=a*x*=*a*.

***5 points***

Suppose f(x)=x+5x−3*f*(*x*)=*x*−3*x*+5​ and g(x)=x2−1*g*(*x*)=*x*2−1​ are functions on their respective domains. Which of the following statements are correct?

The domain of the composite function (f∘g)(x)(*f*∘*g*)(*x*) is (−∞,−10)∪(−10,−1]∪[1,10)∪(10,∞)(−∞,−10​)∪(−10​,−1]∪[1,10​)∪(10​,∞).

The domain of the composite function (f∘g)(x)(*f*∘*g*)(*x*) is R∖{−10,10}R∖{−10​,10​}.

(f∘g)(x)=x2−1+5x2−1−3(*f*∘*g*)(*x*)=*x*2−1​−3*x*2−1​+5​.

(g∘f)(x)=4x+1∣x−3∣(*g*∘*f*)(*x*)=∣*x*−3∣4*x*+1​​.

***5 points***

Consider a sequence {an*an*​} defined as an=an−1+an−22*an*​=2*an*−1​+*an*−2​​ for all n≥3*n*≥3 and a1=0,a2=1*a*1​=0,*a*2​=1. Which of the following statements are correct?

The sequence {an}{*an*​} is not convergent.

lim⁡n→∞an=23*n*→∞lim​*an*​=32​.

∑i=3nai=a2+an−12+∑i=3n−2ai*i*=3∑*n*​*ai*​=2*a*2​+*an*−1​​+*i*=3∑*n*−2​*ai*​.

∑i=3nai=an−12+∑i=2n−2ai*i*=3∑*n*​*ai*​=2*an*−1​​+*i*=2∑*n*−2​*ai*​.

Stock price (y*y*) (in ₹₹) for a motor cycle company (A)(*A*) is predicted by the equation  
                   y=−7log2(x+a)+35,*y*=−7*log*2​(*x*+*a*)+35,  
  
where x*x* represents the number of months since January of the year 2022 (note: for January, consider x*x*= 0) and a∈N*a*∈N. If the stock price of the company goes to zero in November of the year 2022, following the same trend, then find the value of a*a*.

***4 points***

Ravi borrowed ₹₹ 3,000 and ₹₹ 12,000 from his friends Vinay and Bhumi respectively. Vinay lent the money at 7 percent simple interest per annum for 4 years and Bhumi lent the money at 10 percent compound interest per annum for x*x* years. The compound interest which Bhumi received after x*x* years is thrice the value of the simple interest which Vinay received after 4 years. What is the value of x*x*?  
[Note: Simple interest = PTR100100*PTR*​ and Compound Interest = P(1+R100)T−P*P*(1+100*R*​)*T*−*P*, where P*P* is the principle amount, T*T* is time (in years) and R*R* is the interest rate per annum, i.e., if x%*x*% is the interest rate per annum then R=x*R*=*x*]

***5 points***

**Use the following information for questions 7-9:**  
Consider the function defined as follows with p,q,r∈R*p*,*q*,*r*∈R:  
f(x)={pex−4x+3if x < 0 q−5if x = 0rsin(x)+9cos(x)ifx > 0*f*(*x*)=⎩⎨⎧​*pex*−4*x*+3*q*−5*rsin*(*x*)+9*cos*(*x*)​if x < 0 if x = 0ifx > 0​

If the limit exists at x=0*x*=0 for the given function f(x)*f*(*x*), then what will be the value of p*p*?

***2 points***

If f*f* is continuous at x=0*x*=0, then find the value of q22*q*​.

***3 points***

If f*f* is differentiable everywhere, then find the value of r*r*.

***4 points***

**Use the following information for questions 10-11:**  
  
Suppose f*f* is a real valued function defined on domain D*D*. let f(x+y)=f(x)f(y)*f*(*x*+*y*)=*f*(*x*)*f*(*y*) for all x,y∈D*x*,*y*∈*D* and f(1)=5*f*(1)=5, f′(0)=3*f*′(0)=3.

What is the value of f(0)*f*(0)?

***2 points***

What is the value of f′(1)*f*′(1)?

***3 points***

**Use the following information for questions 12-14:** Consider a sequence {an}{*an*​} defined as an={3n−⌊n2⌋7+nwhen n is odd5n2−4n+16n+2n2when nis even *an*​={7+*n*3*n*−⌊2*n*​⌋​6*n*+2*n*25*n*2−4*n*+1​​when n is oddwhen nis even ​  
where ⌊x⌋⌊*x*⌋ is the greatest integer that is less than or equal to a real number x*x*

***2 points***

Which of the following statements are correct?

If n*n* is odd, then ⌊n2⌋=n−12⌊2*n*​⌋=2*n*−1​.

If n*n* is even, then ⌊n2⌋=n2+1⌊2*n*​⌋=2*n*​+1.

If n*n* is odd, then ⌊n2⌋=n+12⌊2*n*​⌋=2*n*+1​.

If n*n* is even, then ⌊n2⌋=n2⌊2*n*​⌋=2*n*​.

Find the limit of the sequence {4an}{4*an*​}.

***3 points***

Find the limit of the sequence {bn}{*bn*​} defined as bn=4an2−10an*bn*​=4*an*2​−10*an*​.